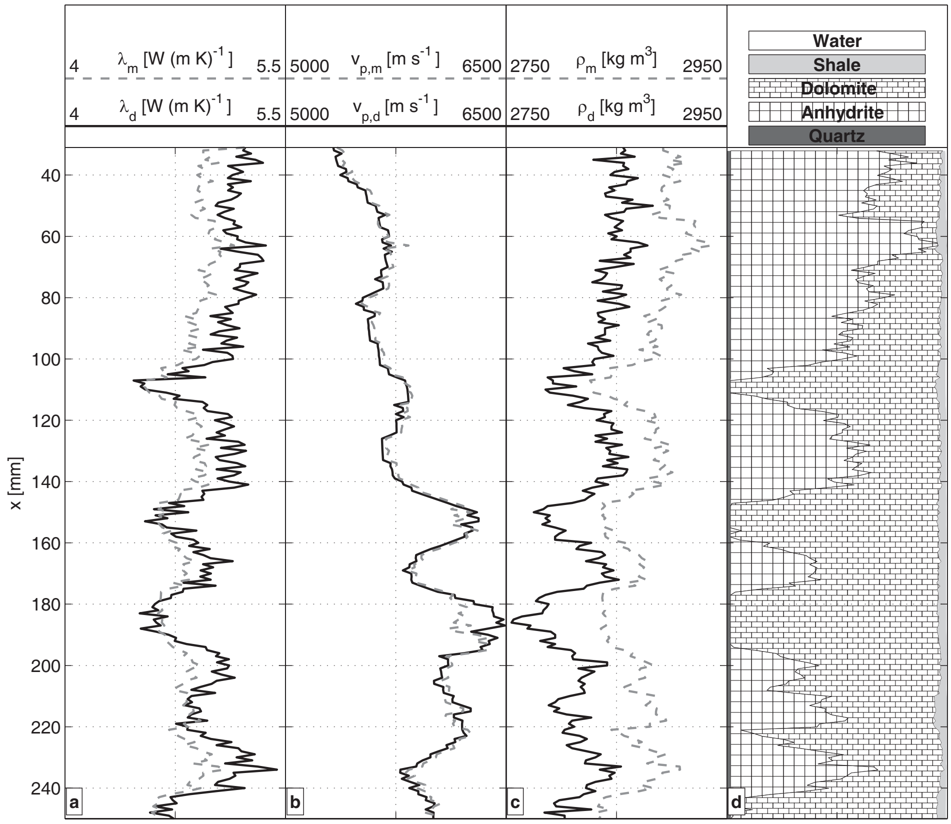

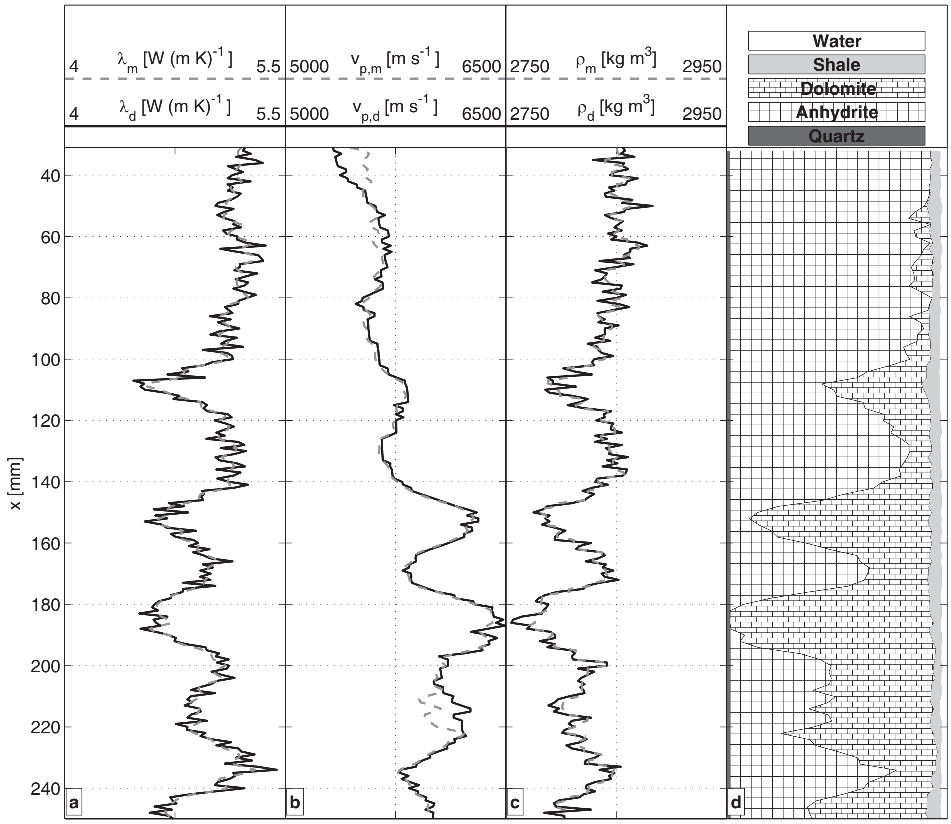

Abbildung F.18: *Results of the inversion of high resolution measurements for the anhydrite/dolomite sample.* Top: *Results for model 1.* Bottom: *Results for model 6.* a) – c) *Measured data and computed values for thermal conductivity, acoustic velocity, and bulk density;* d) *Mineral composition of the sample.*

be avoided. In conjunction with the regularisation of the GSTH amplitudes, this will always lead to dampened version.

This seemingly negative quality turns out to be quite useful in order to investigate the actual information content of a set of data. It would of course be very desirable to get an accurate estimate of the GSTH, but the truth is that this information cannot be safely inferred from the data set. A more simple model that doesn't use all information sources available might give a false impression of the actual data quality and the inference possible. This is also demonstrated in the second, purely petrophysical, example. The different models with varying parameters and complexity constrain the range of plausible models and also point to weaknesses in the data set. In general, these are the mineral properties of the petrophysical model. Such values are tabulated but often need to be adjusted in the analysis process to get a result consistent with the input data.

Appendix: Mathematical derivations

Generalised t-th order mean

The generalised t-th order mean λ_t

$$\lambda_t(\phi, \lambda_p, \lambda_m, t) = \left[\phi\lambda_p^t + (1-\phi)\lambda_m^t\right]^{1/t} \tag{F.36}$$

reduces to the harmonic mean, geometric mean, and arithmetic mean for values of $t = -1, 0, 1$, respectively. For $t = -1$ and 1 this can be seen easily by inserting the values into equation F.36. To obtain the geometric mixing law, however, the limit for $t \to 0$ has to be computed,

$$\lambda_{t=0} = \lim_{t\to 0} \lambda_t = \lim_{t\to 0}[\phi\lambda_p^t + (1-\phi)\lambda_m^t]^{1/t}. \tag{F.37}$$

This equation is transformed by taking the natural logarithm of both sides and using the fact that $\log(\lim_{x\to 0} f(x)) = \lim_{x\to 0} \log(f(x))$

$$\log\lambda_{t=0} = \lim_{t\to 0} \log\lambda_t = \lim_{t\to 0} \frac{\log[\phi\lambda_p^t + (1-\phi)\lambda_m^t]}{t}. \tag{F.38}$$

Using the rule of de l'Hospital yields:

$$\log\lambda_{t=0} = \lim_{t\to 0} \frac{\phi\lambda_p^t \log\lambda_p + (1-\phi)\lambda_m^t \log\lambda_m}{\phi\lambda_p^t + (1-\phi)\lambda_m^t}, \tag{F.39}$$

$$= \phi\log\lambda_p + (1-\phi)\log\lambda_m. \tag{F.40}$$

Exponentiating both sides of this expression gives the familiar geometric mixing law.

Gamma-ray tool

Inserting the impulse response of the tool (equation F.5) into the integral equation F.7 gives

$$GR(z_d) = \sum_l \frac{GR_l}{2\alpha} \int_{z_{l-1}}^{z_l} \mathrm{e}^{-|z'-z_d|/\alpha} dz'. \tag{F.41}$$

The integral in this equation has to be solved separately for the three cases where $(z_d \leq z_{l-1})$, $(z_d \geq z_l)$, and $(z_{l-1} < z_d < z_l)$. For case 1 we obtain

$$
\begin{aligned}
z_d \leq z_{l-1}: \quad & \frac{GR_l}{2\alpha} \int_{z_{l-1}}^{z_l} \exp\left(-\frac{z' - z_d}{\alpha}\right) dz'. && \text{(F.42)} \\
&= \frac{GR_l}{2\alpha} \left[-\alpha \exp\left(-\frac{z' - z_d}{\alpha}\right)\right]_{z_{l-1}}^{z_l} && \text{(F.43)} \\
&= -\frac{GR_l}{2} \left[\exp\left(-\frac{z_l - z_d}{\alpha}\right) - \exp\left(-\frac{z_{l-1} - z_d}{\alpha}\right)\right] && \text{(F.44)} \\
&= \frac{GR_l}{2} \left|\exp\left(-\frac{|z_l - z_d|}{\alpha}\right) - \exp\left(-\frac{|z_{l-1} - z_d|}{\alpha}\right)\right| && \text{(F.45)}
\end{aligned}
$$

Case 2:

$$
\begin{aligned}
z_d \geq z_{l-1}: \quad & \frac{GR_l}{2\alpha} \int_{z_{l-1}}^{z_l} \exp\left(\frac{z' - z_d}{\alpha}\right) dz'. && \text{(F.46)} \\
&= \frac{GR_l}{2\alpha} \left[\alpha \exp\left(\frac{z' - z_d}{\alpha}\right)\right]_{z_{l-1}}^{z_l} && \text{(F.47)} \\
&= \frac{GR_l}{2} \left[\exp\left(\frac{z_l - z_d}{\alpha}\right) - \exp\left(\frac{z_{l-1} - z_d}{\alpha}\right)\right] && \text{(F.48)} \\
&= \frac{GR_l}{2} \left|\exp\left(-\frac{|z_l - z_d|}{\alpha}\right) - \exp\left(-\frac{|z_{l-1} - z_d|}{\alpha}\right)\right| && \text{(F.49)}
\end{aligned}
$$

which is the same result as for case 1. Case 3 gives rise to the following equations:

$$
\begin{aligned}
& \frac{GR_l}{2\alpha} \int_{z_{l-1}}^{z_l} \exp\left(\frac{z' - z_d}{\alpha}\right) dz'. && \text{(F.50)} \\
=\ & \frac{GR_l}{2\alpha} \int_{z_{l-1}}^{z_d} \exp\left(-\frac{z' - z_d}{\alpha}\right) + \frac{GR_l}{2\alpha} \int_{z_d}^{z_l} \exp\left(\frac{z' - z_d}{\alpha}\right) && \text{(F.51)}
\end{aligned}
$$

With the solutions of case 1 and 2 this equations becomes

$$
\begin{aligned}
& \frac{GR_l}{2} \left|1 - \exp\left(-\frac{|z_{l-1} - z_d|}{\alpha}\right)\right| + \frac{GR_l}{2} \left|\exp\left(-\frac{|z_l - z_d|}{\alpha}\right) - 1\right| && \text{(F.52)} \\
=\ & GR_l \left[1 - \frac{1}{2}\exp\left(-\frac{|z_{l-1} - z_d|}{\alpha}\right) - \frac{1}{2}\exp\left(-\frac{|z_l - z_d|}{\alpha}\right)\right] && \text{(F.53)}
\end{aligned}
$$

When the layer k is defined by $z_{k-1} < z_d < z_k$ the combined solution is given by:

$$
\begin{aligned}
GR(z_d) &= \sum_{l, l \neq k} \frac{GR_l}{2} \left|\exp\left(-\frac{|z_l - z_d|}{\alpha}\right) - \exp\left(-\frac{|z_{l-1} - z_d|}{\alpha}\right)\right| \\
&+ GR_k \left[1 - \frac{1}{2}\exp\left(-\frac{|z_{k-1} - z_d|}{\alpha}\right) - \frac{1}{2}\exp\left(-\frac{|z_k - z_d|}{\alpha}\right)\right] && \text{(F.54)}
\end{aligned}
$$

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